

Center Dominance Recovered: Direct Laplacian Center Gauge*

Manfried Faber^a, Jeff Greensite^b and Štefan Olejník^c

^aAtominstytut der österreichischen Universitäten, Tech. Univ. Wien, A-1040 Vienna, Austria

^bPhysics and Astronomy Dept., San Francisco State Univ., San Francisco, CA 94117, USA

^cInstitute of Physics, Slovak Academy of Sciences, SK-842 28 Bratislava, Slovakia

We introduce a variation of direct maximal center gauge fixing: the “direct Laplacian” center gauge. The new procedure consists of first fixing to the Laplacian adjoint Landau gauge, followed by overrelaxation to the nearby Gribov copy of the direct maximal center gauge. Certain shortcomings of maximal center gauge, associated with Gribov copies, are overcome in the new gauge, in particular center dominance is recovered.

1. DRAMA OF GRIBOV COPIES: THE FIRST FIVE ACTS

We proposed a procedure for identifying center vortices in lattice configurations based on center projection in maximal center (or adjoint Landau) gauge, and accumulated evidence in favor of the center vortex model of color confinement [1]. MCG fixing, however, suffers from the Gribov copy problem: The iterative gauge fixing procedure converges to a local maximum which is slightly different for every gauge copy of a given lattice configuration. The problem seemed quite innocuous at first; we observed that vortex locations in random copies of a given configuration are strongly correlated. Recently, the successes of the approach based on MCG fixing have been overshadowed by serious difficulties, so serious that they led the authors of Ref. [2] to speak about the *Gribov copy drama*. Indeed, it follows the structure of the classical ancient tragedy:

1. *Exposition*: The projected string tension in MCG was shown to reproduce the full asymptotic string tension (*center dominance*) [1].

2. *Complication*: Kovács and Tomboulis [3] observed that if we fix to *Landau gauge*, before overrelaxation to MCG, center dominance is lost!

3. *Climax*: Bornyakov et al. [4] pointed out strong dependence of results on the number of gauge copies used in MCG maximization. They chose the best

from N_{cop} copies and extrapolated for $N_{cop} \rightarrow \infty$: the full string tension was underestimated by about 30%!

4. *Reversal*: It was shown [5] that the results depend on the number of gauge copies *and the lattice size*. On a large lattice (compared to the typical size of the vortex core) the problem disappears.

5. *Catastrophe*: Bornyakov et al. [2] observed that using *simulated annealing* one can find better maxima than with the usual method of *overrelaxation*, but the center projected string tension again is only about 2/3 of the full string tension.

Our aim in this paper is to argue that, unlike ancient tragedies, the Gribov copy drama can have the sixth act, a happy ending. We shall introduce a new procedure, direct Laplacian center gauge (DLCG), that overcomes the difficulties associated with Gribov copies. Before doing that, however, we shall briefly review a new insight into MCG fixing, due to [6,7].

2. GAUGE FIXING AS FINDING A “BEST FIT”

Imagine running a Monte Carlo simulation and asking for the pure gauge configuration closest, in configuration space, to the given thermalized lattice. It is easy to show that finding such an “optimal” configuration is equivalent to fixing to the *Landau gauge*.

Allow now for Z_2 dislocations in the gauge transformation, i.e. fit the lattice configuration by one with thin center vortices:

$$U_\mu^{vor}(x) \equiv g(x)Z_\mu(x)g^\dagger(x+\hat{\mu}), \quad Z_\mu(x) = \pm 1. \quad (1)$$

Such $U_\mu^{vor}(x)$ is a continuous pure gauge in the ad-

*Presented by Š. Olejník. Supported in part by the NATO Collaborative Linkage Grant No. PST.CLG.976987 and by the Slovak Grant Agency for Science (Grant VEGA No. 2/7119/2000).

joint representation, which is blind to the $Z_\mu(x)$ factor. Then the fit can be easily performed in two steps:

1. one can determine $g(x)$ up to a Z_2 transformation by minimizing the square distance between $U_{A\mu}$ and $U_{A\mu}^{vor}$ in the adjoint representation – this is equivalent to fixing to the adjoint Landau gauge, aka *direct maximal center gauge*;

2. $Z_\mu(x)$ is then fixed by the *center projection prescription*: $Z_\mu(x) = \text{sign Tr} [g^\dagger(x) U_\mu(x) g(x + \hat{\mu})]$.

It is now immediately clear that $U_\mu^{vor}(x)$ is a bad fit to $U_\mu(x)$ at links belonging to thin vortices (i.e. to the P-plaquettes formed from $Z_\mu(x)$). We recall that a plaquette p is a P-plaquette iff $Z(p) = -1$ (where $Z(C)$ denotes the product of $Z_\mu(x)$ around the contour C) and that P-plaquettes belong to P-vortices. Let us write the gauge transformed configuration as

$$^g U_\mu(x) = Z_\mu(x) e^{iA_\mu(x)}, \quad \text{Tr } e^{iA_\mu(x)} \geq 0. \quad (2)$$

At large β values $\frac{1}{2} \text{Tr}[U_P] = 1 - O(\frac{1}{\beta})$, and equals to

$$(Z_P) \frac{1}{2} \text{Tr} \prod_P e^{iA_\mu(x)} \underset{\text{on P-plaq.}}{=} (-1) \times \frac{1}{2} \text{Tr} \prod_P e^{iA_\mu(x)}. \quad (3)$$

The last equation implies that at least at one link belonging to the P-plaquette $A_\mu(x)$ cannot be small, therefore $^g U_\mu(x)$ must strongly deviate from the center element. This indicates that the quest for the global maximum may not be the best strategy; one should rather try to exclude contributions from P-plaquettes where the fit is inevitably bad [7], or modify the gauge fixing procedure to soften the fit at vortex cores.

3. DIRECT LAPLACIAN CENTER GAUGE

Our new proposal to overcome the Gribov copy problem was inspired by the Laplacian Landau gauge [8]. The idea is the following (for details see [9]):

To find the “best fit” to a lattice configuration by a thin center vortex configuration one looks for a matrix $M(x)$ maximizing the expression:

$$R_M = \sum_{x,\mu} \text{Tr} [M^T(x) U_{A\mu}(x) M(x + \hat{\mu})], \quad (4)$$

with a constraint that $M(x)$ should be an $\text{SO}(3)$ matrix in any site x . We soften the orthogonality constraint by demanding it only “on average”: $\langle M^T \cdot M \rangle \equiv \frac{1}{V} \sum_x M^T(x) \cdot M(x) = \mathbf{1}$.

It is convenient to write the columns of $M(x)$ as a set of 3-vectors $\vec{f}_a(x)$: $f_a^b(x) = M_{ab}(x)$. The optimal

$M(x)$ is determined by *three lowest eigenvectors* $\vec{f}_j(x)$ of the covariant adjoint Laplacian operator:

$$D_{ij}(x, y) = 2D\delta_{xy}\delta_{ij} - \sum_{\pm\mu} [U_{A,\pm\mu}(x)]_{ij} \delta_{y, x \pm \hat{\mu}}. \quad (5)$$

The resulting real matrix field $M(x)$ has further to be mapped onto an $\text{SO}(3)$ -valued field $g_A(x)$. A *naive map* (which could also be called *Laplacian adjoint Landau gauge*) amounts to choosing $g_A(x)$ closest to $M(x)$. Such a map is well known in matrix theory and is called *polar decomposition*.

A better procedure, in our opinion, is the *Laplacian map*, that leads to *direct Laplacian center gauge*. We try to locate $g_A(x)$ as close to $M(x)$ local maximum of the MCG (constrained) maximization problem. To achieve this, we first make the naive map (polar decomposition), then use the usual quenched maximization (overrelaxation) to relax to the nearest (or at least nearby) maximum of the MCG fixing condition.

4. CENTER DOMINANCE

To test the procedure of the last section, we have recalculated the vortex observables introduced in our previous work [1], with P-vortices located via center projection after fixing the lattice to the new direct Laplacian center gauge. The full set of results has been published in [9]. Here we just concentrate on the issue of center dominance which is crucial for the whole picture. Without being able to reproduce the string tension of the full theory, one cannot claim to have isolated degrees of freedom that are associated with the confinement mechanism.

A subset of our results, obtained at a variety of couplings on our largest lattice sizes, is shown in Fig. 1. It is clearly seen that center dominance is restored in DLCG. Moreover, we once again observe precocious linearity (the very weak dependence of projected Creutz ratios on the distance R) signalling that the center projected degrees of freedom have isolated the long range physics, and are not mixed up with ultraviolet fluctuations. Creutz ratios differ by at most about 10% from the phenomenological value of the string tension.

Also other physical results are reproduced in the new gauge: scaling of the vortex density, agreement of ratios of vortex limited Wilson loops with simple expectations, the fact that removing center vortices from lattice configurations destroys confinement [9].

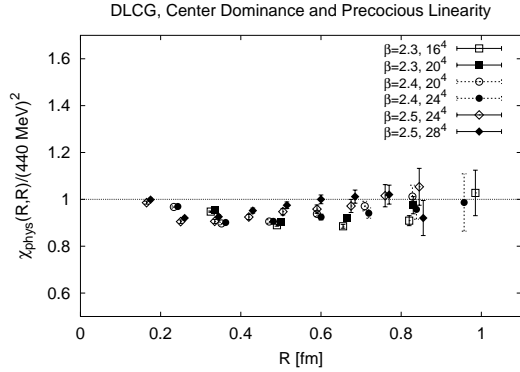


Figure 1. The ratio of projected Creutz ratios to the full asymptotic string tension, as a function of R .

5. COMPARISON TO INDIRECT LCG

To avoid problems with Gribov copies, de Forcrand et al. [10] proposed the Laplacian center gauge. The procedure is similar to ours, but differs in a few important points. The gauge transformation matrix is found from *two* lowest eigenvectors of the covariant adjoint Laplacian operator, one first fixes $SU(2)$ to the $U(1)$ subgroup, and center vortex surfaces should in principle be identified via ambiguities of the gauge fixing condition. However, there is no good separation between confinement and short range physics in this gauge (center dominance seen only at largest distances, no precocious linearity, vortex density not scaling), and identification of vortices on a lattice via gauge fixing ambiguities is practically impossible (center projection is necessary). Langfeld et al. [11] therefore used overrelaxation to MCG after fixing to LCG. This we would call *indirect LCG* (ILCG).

It turns out that results from DLCG and ILCG are quite similar. Center dominance, precocious linearity, and vortex density scaling are observed, however, Creutz ratios in ILCG are somewhat lower than those in DLCG. One can attribute the similarity of both procedures to the strong correlation between P-vortex locations in both gauges. This is illustrated in Fig. 2 showing Creutz ratios calculated from “product Wilson loops” $W_{prod}(C) = \langle Z_{ILCG}(C) Z_{DLCG}(C) \rangle$. If there were no correlation, these ratios should approach twice the asymptotic string tension, while they should go to zero for strong correlation. Our data clearly favor the latter case.

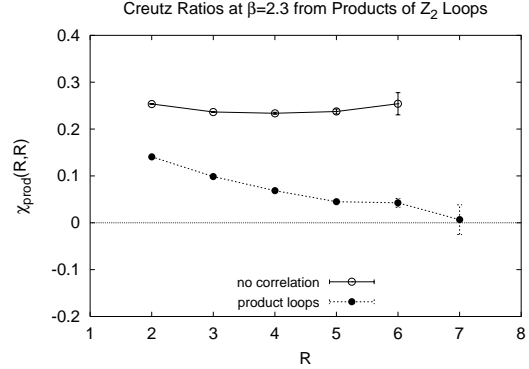


Figure 2. Creutz ratios from “product Wilson loops” show strong correlation between DLCG and ILCG.

6. CONCLUSION

We have proposed a new gauge, DLCG, that combines fixing to Laplacian adjoint Landau gauge with the usual quenched maximization. The first step of the procedure is unique, in the second step no strong gauge copy dependence appears. The procedure can be interpreted as a “best fit” of a lattice configuration by thin vortices, softened at vortex cores. Center dominance, precocious linearity and scaling of the vortex density are recovered in the new gauge.

REFERENCES

1. L. Del Debbio, M. Faber, J. Giedt, J. Greensite, and Š. Olejník, Phys. Rev. D58 (1998) 094501.
2. V. Bornyakov et al., Phys. Lett. B497 (2001) 151.
3. T. Kovács and E. Tomboulis, Phys. Lett. B463 (1999) 104.
4. V. Bornyakov et al., JETP Lett. 71 (2000) 231.
5. R. Bertle, M. Faber, J. Greensite, and Š. Olejník, JHEP 0010 (2000) 007.
6. M. Engelhardt and H. Reinhardt, Nucl. Phys. B567 (2000) 249.
7. M. Faber, J. Greensite and Š. Olejník, Phys. Rev. D64 (2001) 034511.
8. J. Vink and U. Wiese, Phys. Lett. B289 (1992) 122.
9. M. Faber, J. Greensite and Š. Olejník, hep-lat/0106017.
10. C. Alexandrou, M. D’Elia and Ph. de Forcrand, Nucl. Phys. Proc. Suppl. 83–84 (2000) 437.
11. K. Langfeld et al., Phys. Lett. B504 (2001) 338.